LITERATURE CITED

1. E. U. Repik and V. K. Kuzenkov, *Inzh.-Fiz. Zh.*, 57, No. 6, 913-917 (1989).
2. R. E. Rayle, Paper, *Am. Soc. Mech. Eng.*, No. A-234 (1959).
3. Yu. M. Bychkov, *Visualization of Thin Flows of an Incompressible Fluid* [in Russian], Kishinev (1980).

DETERMINATION OF DISTRIBUTION FUNCTIONS FOR THE CHARACTERISTICS OF HEAT AND MASS EXCHANGE PROCESSES BY MEANS OF BOLOMETRIC CONVERTERS

A. B. Katrich and V. V. Kamyshan

UDC 536.5:621:317.794

We propose a method for reconstructing the distribution functions of temperature over the cross section of a flow on the basis of data derived from bolometers. We have optimized the algorithm based on approximation of the distribution functions by bicubic splines with subsequent pseudoconversion of the matrix equation.

The effectiveness with which bolometric converters can be utilized in the diagnostics of heat and mass exchange processes is associated with the high stability of these bolometers to thermal and chemical actions, to their mechanical strength, and to the simplicity of recording the response. For the moment, the basic area for the use of such converters is the measurement of integral characteristics, i.e., the average velocity or temperature of a flow, the total energy or power of radiation fluxes or streams of charged particles. For example, in order to measure the energy of laser emission, a converter has the form of a grid of mutually parallel cylindrical bolometers positioned perpendicular to the radiation flux, and these bolometers are connected in series to a device which recorded the change in grid resistance [1]. With the aid of a similar design it is possible to measure the temperature of a stream of gas or its velocity on the basis of the reduction in resistance in the bolometers heated by the current.

An important feature of these measuring converters (as described here) is the insignificant interaction with the flow being diagnosed, and this is proportional to the ratio of the bolometer diameter to the distance between them (the spacing of the grid), as well as the possibility of using such converters in flows of large cross section. In these energy measuring devices [1] the losses are on the order of 10^{-2}-$10^{-3}$with a flow diameter of up to one meter. With a reduction in the bolometer diameter the time constant of the response is also reduced (about 5 msec for platinum bolometers 10 μm in diameter) and it becomes possible to record high-speed processes.

However, in the integrated diagnosis of heat and mass exchange processes it becomes necessary to measure the distribution functions of the physical quantities through the cross section of the flow. It is not through measurement of the total resistance [1] that additional information on the utilization of bolometric grid converters can be achieved, but rather from the increment in the resistance for each of the grid bolometers. The derived totality of signals represents a projection of the temperature distribution function in the direction of the bolometer axes. In order to obtain the actual distribution it becomes necessary to solve an inverse problem of computational diagnostics, namely, to reproduce the sought function from the set of its projections (or affects) [2]. A unique feature of this problem is the requirement that we introduce the smallest possible number of perturbations, i.e., to optimize the number of projections and the bolometers within them. Estimates show that in real situations the number of bolometer grids cannot exceed 10.

Under the limited aspect conditions the existing a priori information is normally inadequate for effective application of the methods of integral transformation. The method involving the expansion of the sought function f(x) over the basis of some finite-dimensional space $\{S_k\}_1^N$, $x = (x_1, x_2)$ is therefore fundamental:

Gor'kii State University, Kharkov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 59, No. 2, pp. 208-211, August, 1990. Original article submitted January 16, 1989.

 0022-0841/90/5902-0974$12.50

$$f_Y(x) = \sum_{k=1}^{N} y_k S_k(x). \tag{1}$$

After substitution of (1) into the original integral equation we derive the following system of linear algebraic equations:

$$AY = F \tag{2}$$

with a rectangular projection matrix A. Here Y is the vector of the sought expansion coefficients, while the right-hand side F is the vector of the measured values for the signals from the bolometric converters. The problem of reproducing f(x) is substantially incorrect, owing to the angular and radial discreteness and measurement error both in the original formulation and in the algebraic form (2). The fundamental problem, therefore, involves selection of the algorithm and the construction of a limitational functional such that by means of this functional it becomes possible out of the entire set of possible solutions for the integral equation to isolate the correctness sets which contains the solutions most probable from the physical standpoint.

Out of physical considerations the constraint functional must include conditions of nonnegativeness and the limiting constraint on the solution norm. The finiteness of the dimensions and spatial resolution of the converter provide for limiting conditions on the carrier of the function itself and of its spectrum within the limits of error governed by the discreteness. The spatially localized basis functions with a uniformly diminishing spectrum correspond to such a functional.

Basis splines of various orders have been tested in this study, as well as the Gaussians and functions with a limited spectrum, when (1) satisfies the Kotel'nikov theorem. The use of special bases, including orthogonal polynomials, is valid only under the conditions of the hypothecated topological proximity of the basis functions and the possible solutions. Such bases effectively isolate the correctness set, but markedly reduce the universality of the algebraic method.

For $S_k(x)$, localized within the limits of the discreteness interval, the iteration algorithms are optimum in conjunction with a large number of unknowns. Iteration solution of system (2) is accomplished in a parametric analyzer [3], defining the radiation intensity distribution function for technological lasers in a 24 × 24 image element grid. The use of bilinear or bicubic splines [4] makes it possible for the same spatial resolution to reduce the dimensionality of the projection matrix and to apply direct-solution methods to system (2). The existence of a measurement error and the original inaccuracy in the problem do not guarantee compatibility of the system. Therefore, the normal pseudosolution [5] was taken for Y, since it offered the smallest norm and minimized the discrepancy norm in (2). For pseudoconversion, we employed factoring of the form $A = U\Sigma V^T$, so that

$$Y = A^+F. \tag{3}$$

Here A^+ is the pseudoreciprocal A matrix, U and V are the orthogonal matrices; Σ is the diagonal matrix containing singular numbers σ_i in diminishing order. The process of solution consists of successive calculations of the vectors $D = U^TF$, Z with elements $z_i = d_i/\sigma_i$ for $\sigma_i \neq 0$ and $z_i = 0$ for $\sigma_i = 0$, and then the actual solution $Y = VZ$ with calculation of $f_Y(x)$ from (1). Such factoring of the projection matrix makes it possible to study the conditionality of system (2) by analyzing the distribution of the singular numbers.

Figure 1 shows the relationship between the reproduction error

$$\delta_Y = \frac{\|f(x) - f_Y(x)\|_E}{\|f(x)\|_E}$$

for the function $f(x) \neq \sum_{m=1}^{M} \alpha_m \exp[-\beta(x-\gamma_m)^2]$ for $M = 2$, $\beta < 6$, $0 \leq \alpha_m \leq 1$, for a basis of bicubic B splines. The least error in the approximately identical overall number of bolometers corresponds to more uniform discreteness. With a small number of expansion elements N in (1) the discreteness rate is inadequate and the distortions of the spectrum are not offset by the excess number of equations. With an increase in N the approximation error diminishes; however, the computational error associated with the lack of original information, increases and predominates at some particular point.

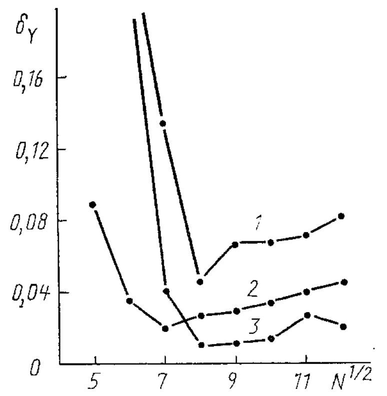

Fig. 1

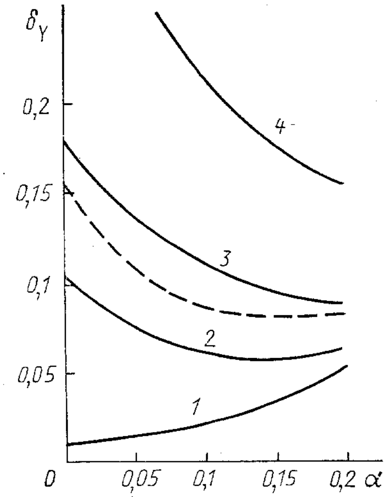

Fig. 2

Fig. 1. Reconstruction errors as a function of discreteness frequency. Curve 1) 4 projections, 13 bolometers, 52 equations; 2) 8, 7, 56; 3) 6, 9, 54.

Fig. 2. Effect of rank limitation. Additive error in original data: 1) 0; 2) 0.03; 3) 0.06; 4) 0.12. The dashed curve represents a multiplicative error of 0.12.

The use of a basis consisting of bilinear B splines yields analogous results, but because of the higher error of approximation the values of δ_Y are somewhat larger.

According to estimates [5, 6], the error in the solution of (2) is proportional to the conditional number $\text{cond}^+A = \sigma_{max}/\sigma_{min}$, where σ_{min} is the least of the singular numbers used in the solution of the system. For all of the matrices A considered here, the singular numbers initially diminish slowly, and then in the transition region their values fall by several orders to a level defined by the calculation error. When using singular numbers in (3) larger than σ_{min} the derived solution will approximate the normal pseudosolution with a relative error on the order of $\alpha = \sigma_{min}/\sigma_{max}$. The rejection, therefore, of numbers $\sigma_i < \sigma_{min}$ slightly increases the computational error in Y, but significantly improves the conditionality of the system. The cutoff level must be matched to the error δ_F in the input data. With additive error in the original data the optimum cutoff level $\alpha \approx 2\delta_F$ is shown in Fig. 2. For multiplicative error the relationship is analogous, but $\delta_Y < \delta_F$ in the range $\alpha \approx 0.05$-0.25.

The more cumbersome factoring operation for the projection matrix A is accomplished preliminarily on a universal computer. In actual work the results of the factoring are utilized in the form of an existing pseudoreciprocal matrix A^+ or its components U^T, Σ^{-1}, V. The storage method is governed by the number of singular numbers taken into consideration, by the high-speed action and volume of the memory in the computer-processed measurement complex.

For the iteration methods a considerable gain is offered by the lowest-order splines as a consequence of a reduction in the extent to which matrix A is filled. In certain converter configurations it is appropriate to calculate the indices and the magnitude of the nonnull elements of the matrix at each interval. In comparison to the pseudoconversion algorithm the expenditure of machine time on the solution is not increased, and the required operative memory volume is significantly reduced. However, owing to the growth in the approximation error and in the sensitivity to the selection of the initial approximation a more careful selection of the specific algorithm is necessary.

Thus, in diagnostics involving the utilization of bolometric converters it is possible to reproduce the temperature distribution function over the cross section of the flow in its approximation through bicubic splines with utilization of a pseudoconversion algorithm of the resulting matrix equation. The error due to discreteness and the reduction in the rank of the matrix system corresponds approximately for this method to the error in the measurement of the original data, thus allowing us to determine the required number of projections and the discreteness rate. Realistically speaking, the reproduction quality for 100 expansion elements is satisfactory with 4-8 grids, each of which contains 6-15 bolometers.

LITERATURE CITED

1. V. M. Kuz'mychev, Yu. M. Latynin, and I. A. Priz, *PTÉ*, No. 2, 190-192 (1974).
2. A. N. Tikhonov, V. Ya. Arsenin, and A. A. Timonov, *Mathematical Problems of Computer Tomography* [in Russian], Moscow (1987).
3. A. B. Katrich and A. V. Khudoshin, *PTÉ*, No. 2, 221 (1988).
4. Yu. S. Zav'yalov, V. I. Kvasov, and V. L. Miroshnichenko, *Spline Function Methods* [in Russian], Moscow (1980).
5. V. V. Voevodin and Yu. A. Kuznetsov, *Matrices and Computations* [in Russian], Moscow (1984).
6. G. H Wilkinson and K. Rainsh, *Handbook of Algol Algorithms* [Russian translation], Moscow (1976).

FLUCTUATIONS IN THE RATE OF FLOW DURING FILTRATION OF POLYMER SOLUTIONS

M. M. Khasanov and I. N. Yagubov

UDC 542.67:541.6

This article demonstrates the possibility of a loss of stability in steady-state regimes involved in the filtration of polymer solutions, and we present also the construction of a mathematical model on the basis of whose analysis we have ascertained the unique features of self-oscillations and stochastic oscillations which arise in the region of instability.

Nonlinear effects in the filtration of non-Newtonian media may lead to a loss of stability in the steady filtration regime [1-4]. We observed such phenomena in a number of laboratory experiments in which we studied the filtration of polyacrylamide (PAA) solutions through a column filled with quartz sand. The permeability of the porous medium with respect to air amounted to $3.1 \cdot 10^{-12}$ m^2. During the course of the experiment the pressures at the inlet and outlet of the column were maintained at constant levels and the flow rate of the fluid being filtered was measured over a rather prolonged period of time. The experiments demonstrated that with small pressure differences a steady flow rate is established. But when some critical pressure difference Δp_* is attained (dependent on the PAA concentration in the solution) the steady filtration regimes lose stability, and we observe unattenuated fluctuations in the flow rate. As an example, Fig. 1 shows the flow rate for PAA with a concentration of 0.075% as a function of time for the case in which $\Delta p = 0.6$ MPa.

The fluctuations in the flow rate are irregular in nature. The level of irregularity (chaos) can be evaluated on the basis of the Hausdorf scale for the curve $Q = Q(t)$. The quantity D is determined [5, 6] during the process of measuring the length l on the curve $Q = Q(t)$ by means of dividers with an opening η. The measurements are started from the origin P_0. Describing a circle of radius η with the center at P_0, we mark the point P_1 at which the curve initially moves out of the circle. The second point P_2 is obtained when the center of the circle is shifted to the point P_1, etc. If $l(\eta)$ is used to denote the length of the resulting broken line $P_0P_1P_2$..., approximately describing the curve, the length of the curve will be [7, 8].

As demonstrated by direct measurement, for the experimental curves $Q = Q(t)$ with not overly small η, $l(\eta) \sim \eta^{-\gamma}$. Consequently, the graph of the functions $Q = Q(t)$ can be assumed to be fractal curves having the dimension $D = \gamma + 1$. It is natural to assume that the larger the dimension of the experimental curve, the less orderly the process whose image is represented by this curve. Thus, for the curve in Fig. 1 we have $D = 1.40$. We should take note of the fact that after establishment of the chaotic filtration regime any further increase in the pressure difference will not lead to an increase, but rather to a decrease in the Hausdorf dimension for the curves $Q = Q(t)$, which gives evidence of the more orderly progress of the filtration process in the case of larger values of Δp.

Ufa Petroleum Institute, Azizbekov Institute of Petroleum and Chemistry, Azerbaidzhan. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 59, No. 2, pp. 211-215, August, 1990. Original article submitted July 28, 1989.